A Biologist's Guide to Impact Factors

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Abstract

Personal impact factors (e.g., the *h*-index) are becoming more and more important in evaluations of faculty with respect to job hiring, promotion, and tenure, but they are largely poorly understood by the community at large. The purpose of this study is to educate biologist and other scientists about some of the wide literature about impact factors, including highlighting their strengths and weaknesses. This includes a thorough exploration of dozens of such indices by comparing how they perform through repeated calculation of data representing 15 years of scientific output of a single individual from beginning through mid-career. Indices are examined with respect to factors such as interpretability, consistency, and stability.

Introduction

Personal impact factors have become very popular since their original online introduction by Hirsch (later published as Hirsch 2005). Impact factors suggest a means of evaluating the productivity of a researcher beyond simply counting publications or citations by combining both into a single measure. Many scientists have rejected the use of these metrics as misleading or useless since one's impact cannot be captured by a single value, particularly when it comes to hiring and promotion and tenure decisions (Abbott *et al.* 2010). While no one seriously suggests that decisions should be based on a single metric, it has been clear for a long time (prior to the invention of these new measures) that a limited number of metrics will likely be used (Martin 1996), particularly at the administrative level where more nuanced examinations of records is not always possible. In this sense the personal impact factor is being added to "traditional" measures such as publication count and research funding.

Personal impact factors seem to have two primary uses: promotion and tenure decisions and hiring decisions. For promotion and tenure decisions, impact factors may be used as a general measure of researcher quality and impact, where values are compared to some baseline for their field. Determination of the baseline is among the most controversial aspects of these measures since what would be considered a good or bad value is very discipline dependent. One might have very different publishing expectations from a researcher in cellular biology versus a researcher in ecology versus a theoretical biologist, thus in the same way one does not expect the same number of publications from each discipline, one may expect a different measure of impact from each discipline. The context of the decision is also critically important since expectations would be quite different for an assistant professor, an associate professor, or a full professor. While individuals and departments may eschew such metrics, there is a general belief that many administrators are examining these numbers (Abbott *et al.* 2010). I personally started collecting data for my own impact factor calculation after discovering that my Dean's office had incorrectly estimated an impact factor for me during a progress review. It seemed better to provide my own correct data than to trust someone else to do it properly. Generally, academic units need to actively monitor these metrics for their faculty at the time of promotion and tenure, not only to guarantee correct data, but also to provide the proper context of how these factors rate for that researcher's specific discipline. Since this type of context is already required for similar factors such as publication rate and grant funding, adding a similar context for impact factors should hardly be considered a burden and instead needs to be viewed as additional opportunity to make the case for granting a promotion.

The second area impact factors are likely to see use is in separating candidates during the hiring process. These factors are unlikely to be used for final hiring decisions, but may play a role in the early filtering of candidates into a short list for interview. I am unaware of any specific use of these factors in hiring within the biological sciences, but impact factors do appear to have been used during hiring in other disciplines and there is no reason to believe some biological hiring committees will not follow suit. For hiring decisions, the impact factors are not necessarily compared to a baseline but are instead compared amongst candidates as one (but by no means the only) means of ranking.

While many individuals and units have rejected personal impact factors on philosophical grounds (personal observation), in the long run biologists need to care about impact factors because people who make decisions about their careers are going to care about them. For all that impact factors are often derided as meaningless compound values (Abbott *et al.* 2010), in the life-sciences, the *h*-index has been shown to correlate well with number of publications, total citations, average journal impact, and (perhaps most importantly) peer assessments (Bornmann *et al.* 2008b). The more you understand about what a personal impact factor means (and does not mean), the more you can control how it is used to support your own career advancement.

There is a large and rapidly expanding literature on personal impact factors within the fields of scientometrics and bibliometrics which is essentially unknown and invisible to the biological community. The outline of the reminder of this paper is to (1) describe the meaning of the *h*-index, not simply define it; (2) summarize the major weaknesses which have been identified; and (3) briefly discuss the many adaptations and alternative metrics which have been invented to deal with these weaknesses. It should be noted that impact factors such as the *h*-index have also been proposed for many more purposes than simply ranking and rating researchers. Variants of these indices have also been used to compare amongst larger administrative units (centers, departments, universities, and even countries) as well as among research topics, buzzwords, and chemical compounds (Arencibia-Jorge 2009; Arencibia-Jorge *et al.* 2008; Arencibia-Jorge and Rousseau 2009; Bar-Ilan 2010a, b; Bornmann *et al.* 2009a; Lazaridis 2010; Schubert and Glänzel 2007; Schubert *et al.* 2009). This paper is focused on metrics which apply to individuals and will not follow up on these broader institutional or journal measures.

Another important point is that all of the remaining discussion focuses on citation-based metrics. There is a current trend, particularly among the online and open-access community, to develop and focus on impact metrics based on data other than citations (e.g., total-impact.org). This includes things such as page hits, downloads, Twitter mentions, Facebook likes, Google +1’s, Mendeley readers, bookmark counts, etc. While there are some very interesting ideas in the use of these non-traditional types of metrics to measure the impact of a researcher (particularly for content beyond the traditional publication, such as data sets, blogs, etc.), consideration of these sorts of metric is well beyond the scope of this document.

Data

Most impact factors only require knowing the number of citations that each publication has received. Some of the alternates may also require number of authors or the year of the publication (Box 1 lists some basic definitions and symbols which can be used to calculate most of the indices mentioned throughout this paper). A few more complicated metrics may require additional information, which will be described as necessary.

Box 1. Basic definitions

Unless otherwise specified, we will assume that a researcher's publications have been sorted into rank order from most citations to fewest citations.

- P is the total number of publications
- C_i is the number of citations for the i^{th} ranked publication
- N_j is the cumulative number of citations for the first j publications, *i.e.*, $N_j = \sum_{i=1}^{j} C_i$
- N_P is the total number of citations for all publications, *i.e.*, $N_P = \sum_{i=1}^{P} C_i$
- $\bar{C}$ is the average number of citations per publication, *i.e.*, $\bar{C} = N_P / P$
- $C_{Max} = C_1$, the largest number of citations for a single publication
- Y_i is the year that the i^{th} publication was published
- Y_0 is the year of the author's first publication, *i.e.*, the Min(Y_i)
- Y_{Now} is the current year, or more precisely, the year for which the citation data is being calculated (which would be in the past when using older records)
- A_i is the number of authors of the i^{th} publication

The primary sources for citation information tend to be the ISI Citation Index, Scopus and Google Scholar, each of which has advantages and disadvantages (Armbruster 2010; Bar-Ilan 2008; Bornmann *et al.* 2009b; Derrick *et al.* 2010; Franceschet 2010; Harzing In press; Jacsó

2008a, b, c, 2009; Meho and Rogers 2008; Meho and Yang 2007; Mikki 2010; Mingers and Lipitakis 2010). From my own experience, Google Scholar tends to cover a broader range of publications (e.g., theses, books, and more obscure journals) and thus picks up citations that may be missed by ISI, but it also has substantially greater redundancy and duplication issues which can lead to exaggerated citation counts in many instances. As of this writing, Google Scholar has recently gone through some changes which should help reduce some of this redundancy, including the ability for individual scholars to setup profiles which can be corrected for their true publication list. Google Scholar also auto-estimates some basic impact factors, although they may need to be viewed with caution, as there can be strange inconsistencies (e.g., a coauthor recently pointed out that the Google Scholar citation count for a specific coauthored publication is different on her page than on my page). Henziger et al. (2010) recently showed that as long as all data (for comparative purposes) is collected from a single source, the relative ranking of individuals tends to be stable, even when citations or publications may be missing.

Many of the publications about impact factors choose a number of exemplary scholars from a given field as illustration (e.g., Bodman 2010; Franceschet 2010; Harzing In press; Hirsch 2005; Kelly and Jennions 2006; Prathap 2010a; Schreiber 2009). In contrast, I am going to choose data from a single mid-career scholar: myself. (This is not a question of ego or vanity, but rather access to data). Instead of focusing on a single time point, however, I am going to use yearly citation counts starting from 1997 (the year of my first publications) up through 2011.

The data for a specific year is the cumulative sum of citations for everything up through that year (based on the date of the citing publication). Citations which occur in a year prior to the publication of the cited article (i.e., in press citations) are not counted until the year of the actual article publication (e.g., in August 2012 I already knew of two citations for a book chapter which was not being published until 2013. That chapter and those citations would not be counted as part of the 2012 data.

As already discussed, whether observed values are considered good or bad is both field and context dependent; the purpose, instead, is to help illustrate some of the differences, strengths and weaknesses amongst these factors by their stability and properties across time rather than as measured at a single time point. A summary of the data from each time point is shown in Table 1, which contains the more traditional measures of impact (e.g., number of publications and number of citations). The raw citation data which makes up the calculations below was manually curated from multiple sources (primarily the ISI Citation Index, with some supplementation from Google Scholar and other sources) and therefore does not represent figures directly obtained from any specific single database.

All of the results in this paper were calculated with a Python program which can be found at https://github.com/msrosenberg/ImpactFactor.

Table 1. A summary of the data at each time point used to construct the impact factors.

Date	1997	1998	1999	2000	2001	2002	2003	2004
Total Publications	5	6	7	11	15	19	23	26
Total Citations	2	13	36	66	132	239	366	585
Citations per Pub	0.40	2.17	5.14	6.00	8.80	12.58	15.91	22.50
Max Citations	1	6	19	27	42	89	142	218
Date	2005	2006	2007	2008	2009	2010	2011	
Total Publications	31	32	35	35	38	44	48	
Total Citations	845	1176	1509	1891	2314	2770	3191	
Citations per Pub	27.26	36.75	43.11	52.53	59.33	61.56	65.12	
Max Citations	298	378	487	607	750	884	1021	

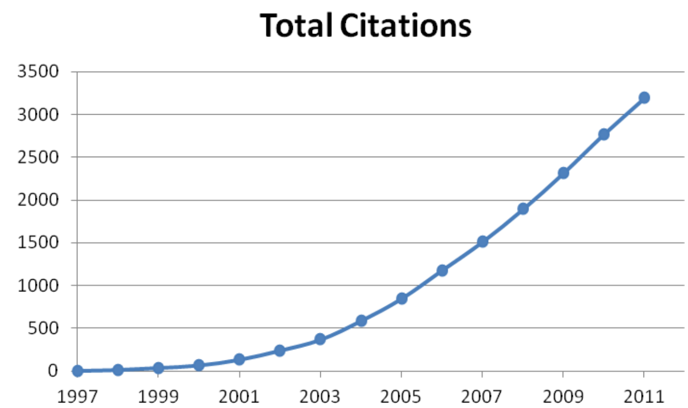

Figure 1. Total citation count through time.

Hirsch's Index

The ***h*-index** (Hirsch 2005) is the most important personal impact factor you need to be familiar with, not because it is necessarily the best, but because (1) it was the first major index of its type and most of the other indices are based on it in some way, and (2) it is the single factor with which most other people you communicate with (e.g., administrators) are likely to be somewhat familiar. You may find another index which you prefer, but everything starts with *h*.

The *h*-index is defined as the largest value for which *h* publications have at least *h* citations. Put another way, a scientist has an impact factor of *h* if *h* of their publications have at least *h* citations and the other $P - h$ publications have $\leq h$ citations. Note that *h* is measured in publications. In formal notation, one might write

$$h = \max_i \left(i \leq C_i \right).$$

The meaning of this index is perhaps best illustrated graphically. Figure 2 shows a theoretical citation distribution curve: a plot of each publication's citation count versus its rank (sometimes called an *h*-graph). The *h*-index is the point where a line through the origin with a slope of one crosses the citation curve. The *h* publications to the left of this point are those that contribute to the *h*-index and are often referred to as the **Hirsch Core** (Rousseau 2006), while the *P* – *h* publications to the right of this point which fall outside of the Hirsch Core are often referred to as the **Hirsch Tail**.

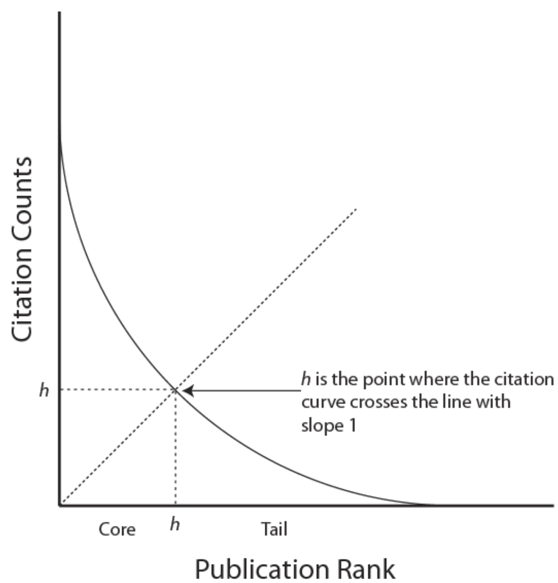

Figure 2. Graphical representation of the h-index (sometimes called an h-graph), including definition of the Hirsch core and tail.

An alternate graphical way of thinking about *h* is that the *h*-index represents the size of the largest square which can tangentially fit under the citation curve. This square divides the citation curve into three sectors: the square itself (the middle sector) which represents the h^2 publications minimally needed to have a score of *h*, the upper sector above the square which represents the excess citations in the core above-and-beyond those necessary to receive a score of *h*, and the lower sector to the right of the square which represents the citations in the tail (citations of the publications outside the core). This illustrates one disadvantage of the *h*-index; it should be readily obvious that citation curves with very different distributions may all encompass the identical square and thus have the same *h*-index. This will be discussed in more detail below.

Using our notation, the total citations within the core would be

$$N_h = \sum_{i=1}^{h} C_i \, .$$

The number of excess citations within the upper sector is thus $N_h - h^2$.

A simple measure of the speed (slope) at which *h* increases over time, **Hirsch's *m*** or the ***m* quotient**, can be estimated simply as the ratio between *h* and the time elapsed since first publication, or

$$m = \frac{h}{Y_{Now} - Y_0} \, ,$$

with its units equal to publications per year.

Table 2. The h-index and related measures for each year.

Date	1997	1998	1999	2000	2001	2002	2003	2004
h-index	1	2	3	5	6	7	9	11
Hirsch-core citations (N_h)	1	9	31	58	120	205	312	507
Hirsch *m*-quotient	n/a	2.00	1.50	1.67	1.50	1.40	1.50	1.57
Date	2005	2006	2007	2008	2009	2010	2011	
h-index	13	15	17	19	23	24	24	
Hirsch-core citations (N_h)	748	1048	1358	1726	2182	2587	2931	
Hirsch *m*-quotient	1.63	1.67	1.70	1.73	1.92	1.85	1.71	

As one would expect, my *h*-index has gradually grown over time. The rate of increase in *h* (the *m*-quotient) has itself increased a bit, generally being around 1.5 for the first half of my career, but having increased to about 1.7 in the second half of career, indicating a slightly acceleration in the change in *h* over time. Over the second half of my career, my *h*-index has approximately doubled while my core citations have approximately quadrupled. To a certain extent, this relationship is expected given that the minimum number of core citations necessary for a specific value of *h* is h^2. Interestingly, the relationship holds true even though the actual number of citations in the core is substantially higher than that minimal number (e.g., $h^2 = 169$ vs. 748 in 2005).

Major Reasons for Variant Indices

Since the original publication of Hirsch's Index there have been dozens of alternate and adapted indices proposed (Van Noorden 2010). Despite the large number of alternatives, most of the reasons for developing the variant metrics fall into a limited number of categories. Before getting into any detail on specific alternatives, each of the main reasons will be briefly discussed since they highlight some of the potential weaknesses of the Hirsch Index and serve as an overview and introduction to the remainder of the discussion.

Redefining the Core

As simple as the Hirsch Index is, not all researchers have felt that it properly captures the scope of impactful publications. A number of variant indices have been proposed which define the core in a different manner. Some of these proposals use a very similar approach to the Hirsch Index, but propose either stricter or looser criteria for including publications in the core. Other methods use fairly different approaches to defining the core and are less directly comparable to the Hirsch method.

Giving Credit for Excess Citations

When an author has a publication whose citation count is well above their *h*-index, additional citations to that publication have essentially no impact on the author's impact factor. A number of metrics have been proposed that give extra weight for excess citations within the core or for all citations for all papers, rather than just the minimal number necessary to reach a specific *h*-index. Generally, these metrics begin with the *h*-index and adjust it for these extra citations.

Describing the Core and Tail

Quite a number of indices have been proposed to describe the citation distribution within the Hirsch core for use in separating authors with identical *h*-indices. These metrics are generally meant to be supplemental to the *h*-index rather than to serve as a replacement. Some focus only on publications and citations which fall within the core, while others compare the contents of the core to those publications and citations in the tail of the citation distribution.

Accounting for Multiple Authors

Should a highly cited publication with a single author be worth the same amount as a highly cited publication with a dozen authors? Many researchers have been concerned with how to give (or remove depending on one's point of view) credit for publications with multiple authors. Generally this involves a two step process. The first step is to define the weight given to an author for a publication. Solo authored publications generally have a weight of one, while multi-authored publications have a weight which ranges between zero and one depending on the method used. If all authors get full credit for the publication, no adjustments for authorship are being made. The most extreme adjustment would be to give only first (or primary depending on the field) author credit for the publication (Hu *et al.* 2010). The simplest approach is to divide credit equally among all authors, although many other approaches are also possible (Galam 2011; Liu and Fang 2012; Wan *et al.* 2007). Once the weight for a publication is determined, the second step is to use the weight to correct that publication. There are two basic approaches to this: one can either correct the number of citations or correct the publication rank (or both). Differences in the various author-corrected approaches depend largely on how one determines authorship credit in the first step and how one uses that credit in the second step.

Accounting for Self-Citations

Many researchers have been concerned that impact factors are biased by self-citation since self-citations do not represent "impact on the field" (e.g., Bartneck and Kokkelmans 2011; Costas *et al.* 2010; Gianoli and Molina-Montenegro 2009; Schreiber 2007, 2008a). A number of counter-arguments have also been made (e.g., Engqvist and Frommen 2008, 2010), such as the fact that highly successful authors should not be punished for citing their own earlier work if the current study builds on previous studies (e.g., reducing redundancy across papers) and that when a researcher's impact factor is high it is not likely to be much affected by self-citation. The most straight-forward approach to accounting for self-citation is to simply remove all self-citations from the citation counts and then calculate the indices using the standard approaches. A complication arises for co-authored papers: should one only remove self-citations from the researcher under investigation or from all coauthors? Because identification of self-citations (particularly when including coauthors) can complicate the data collection process, a few alternative metrics for auto-correcting for self-citations have also been proposed. Servers which attempt to automatically recognize self-citations sometimes define the concept incompletely (Carley *et al.* In press).

Accounting for Time

Many researchers have been concerned with the effect of time on impact factors, although these fall into a number of different areas of concern. The basic indices are career length metrics, which make comparing junior and senior authors difficult. Some metrics explicitly look at reduced time windows (e.g., 5 or 10 year periods) or look at the rate of change of an index through time rather than the value of the index itself. Recently published papers generally have few to no citations since there is generally a lag between publication and the beginning of the citation cycle; some indices attempt to correct for this lag by predicting the number of citations a paper is likely to get over a longer time period. In cases where a time interval needs to be specified as part of the calculation, it is not at all clear what the appropriate time interval should be and this likely varies across fields (Wang In press).

Alternate Impact Factor Indices

The following section briefly describes most of the alternate indices which have been proposed to measure the impact of an author. These are divided by the broad justification for the new metric, as described above. Because many of the metrics have seen little use, a number of symbols have been repeated by different authors (e.g., there are at least three different proposed *w*-indices) leading to a bit of naming ambiguity, but we will endeavor to be as specific as possible while keeping to the original names of the indices. The goal of this section is not to compare formally the performance of these indices under a variety of conditions and assumptions; for such a detailed comparison of many of these factors, see Alonso *et al.* (2009) and Schreiber (2010).

Indices which Redefine the Core

The best known and most widely studied alternate to the h-index is known as the ***g*-index** (Egghe 2006a, b, c). The g-index is designed to give more credit for publication cited in excess of the h threshold (as already mentioned, once a paper has many more citations than the author's h further citations to that paper have essentially no effect on the author's impact as measured by h). The primary difference between the formal definitions of the h- and g-indices is that g is based on cumulative citation counts rather than individual citation counts. Formally, the g-index is the largest value for which g publications have jointly received at least g^2 citations.

$$g = \max_i \left(i^2 \leq N_i \right).$$

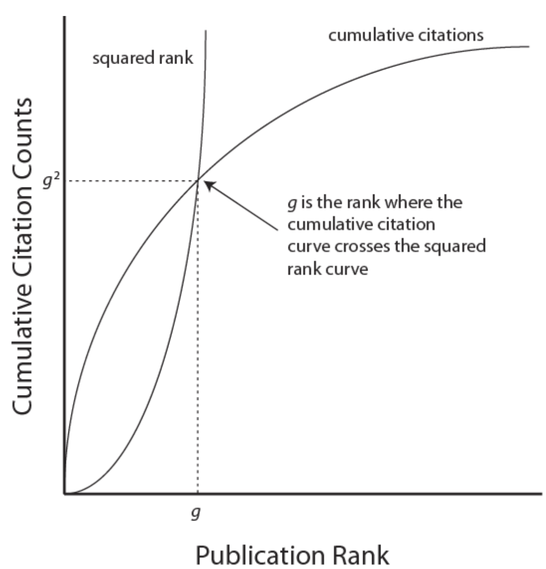

Figure 3. Traditional graphical representation of the g-index.

Graphically, this is the point where the cumulative citation curve (rather than the individual citation curve) crosses the curve of squared rank (Figure 3).

The *g*-index defines a looser criterion for the core than the *h*-index, and it is easily shown that $h \leq g$. One potential problem with *g* is that if the total number of citations is large relative to the number of publications ($N_P > P^2$) these curves will not actually cross (this is currently true for my own citation curve, for example). A few corrections have been suggested, including adding phantom papers with zero citations until they do cross (essentially, this would make *g* equal to the integral value of the square-root of N_P), although I believe a more acceptable solution for this problem is to simply set *g* equal to *P* since, like *h*, *g* is measured in number of publications and the publication impact shouldn't be greater than the total number of publications. As a contrast, based on the 2011 data, using the phantom paper method would give me a *g*-index of 56; restricting the maximum value of *g* to the number of publications gives me a *g*-index of 48.

An alternate interpretation of *g* can be found by rewriting the above equation (Jin 2006) such that

$$g = \max_i \left(i \leq \frac{N_i}{i} \right),$$

which makes it clear that the *g*-index can also be viewed as the largest value for which the top *g* publications average *g* citations. Graphically this is the point where the slope of line one used for the *h*-index crosses the mean citation curve (Figure 4) and allows a much clearer interpretation of *g* than the formal definition.

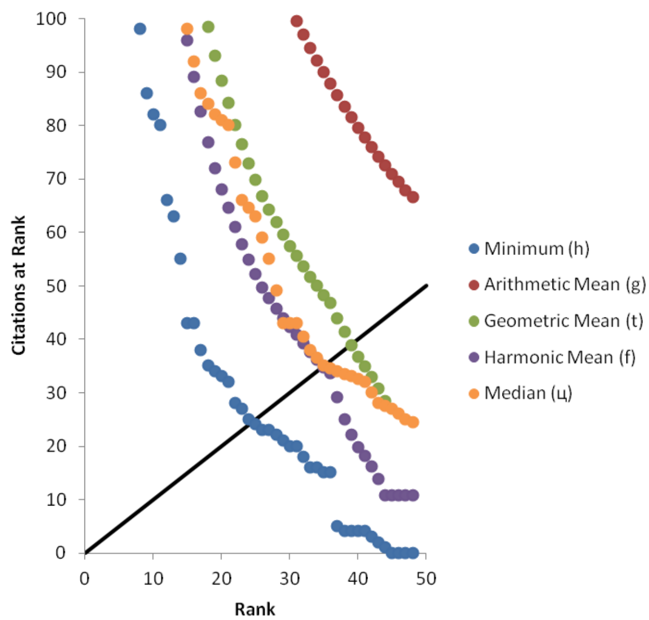

Figure 4. Graphical comparison of the h, g, t, f, and μ-indices for the 2011 data (the left most points of each curve with values above 100 are truncated for clarity). The point where the line of slope 1 crosses each curve indicates the value for that impact factor (note that it does not cross the curve representing g, the arithmetic mean of the citations within the core).

From this standpoint, the *g*-index is based on the arithmetic mean of the citations within the core while *h* is based on the minimum citation count within the core. Tol (2007) proposed two related methods for defining the core, the ***f*-index** and ***t*-index**, which use the harmonic and geometric means of the citations within the core, respectively, rather than the arithmetic mean. Formally, they are calculated as

$$f = \max_k \left(\frac{1}{\frac{1}{k}\sum_{i=i}^{k}\frac{1}{C_i}} \geq k \right) = \max_k \left(\frac{k}{\sum_{i=i}^{k}\frac{1}{C_i}} \geq k \right)$$

and

$$t = \max_k \left(\exp\left(\frac{1}{k} \sum_{i=1}^{k} \ln(C_i) \right) \geq k \right)$$

It is easily shown that $h \leq f \leq t \leq g$. Similarly, Glänzel and Schubert (2010) suggest using the median of the citations within the core, the **μ–index,** and show the μ–index and the *f*-index to be less affected by outliers than the other measures.

Woeginger's *w*-index (Woeginger 2008) is somewhat similar to *h*. It is the largest value of *w* for which publications have at least 1, 2, 3…*w* citations.

$$w = \max_k \left(C_i \geq k - i + 1 \right) \quad \text{for all } i \leq k.$$

This is best interpreted graphically (Figure 5). If the *h*-index describes the largest *h* x *h* square which can fit under the citation curve, Woeginger's *w*-index describes the largest isosceles right-angled triangle (with perpendicular sides of *w* and w) which can fit under the citation curve. It obviously uses a looser criterion for the core than *h* and $h \leq w$.

Unlike the other similar metrics, *w* has the property of defining a core which may contain very low cited publications (e.g., recall that the last paper in the core only requires a single citation). While it does have some interesting properties, Woeginger's *w*-index seems much too liberal to serve as an effective measure of impact and is likely very highly correlated with the total number of publications.

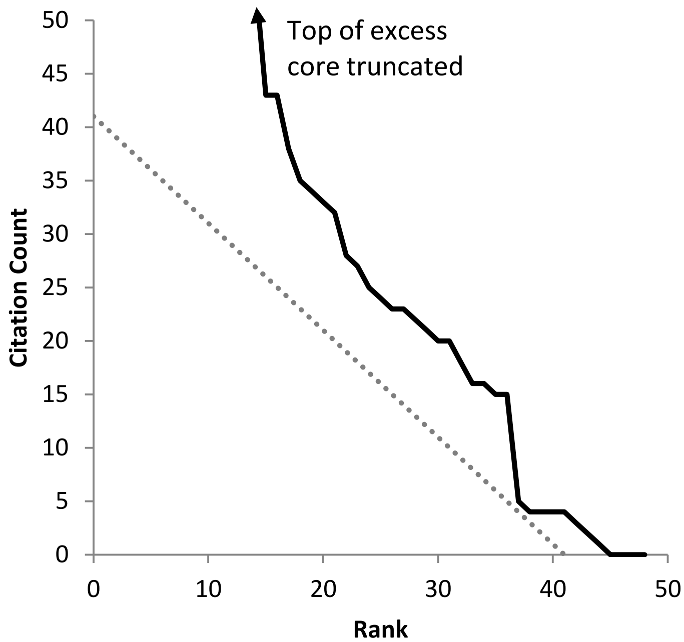

Figure 5. Graphical illustration of Woeginger's w-index. It is the largest isoceles right-angled triangle which can fit under the citation curve. Data from the 2011 citation point.

While the above alternates have more liberal definitions of the core, Kosmulski (2006) proposed the ***h*****(2)-index** to have a stricter definition of the core. This index is the largest value, *h*(2), for which *h*(2) publications have at least $h(2)^2$ citations.

$$h(2) = \max_i \left(i^2 \le c_i \right).$$

This may seem quite similar to the *g*-index, but the *g*-index is based on cumulative citation counts while *h*(2) is based on individual citation counts. While an *h* of 10 would indicate that a scientist had at least 10 publications with 10 citations each, an *h*(2) of 10 would indicate a scientist had at least 10 publications with 100 citations each.

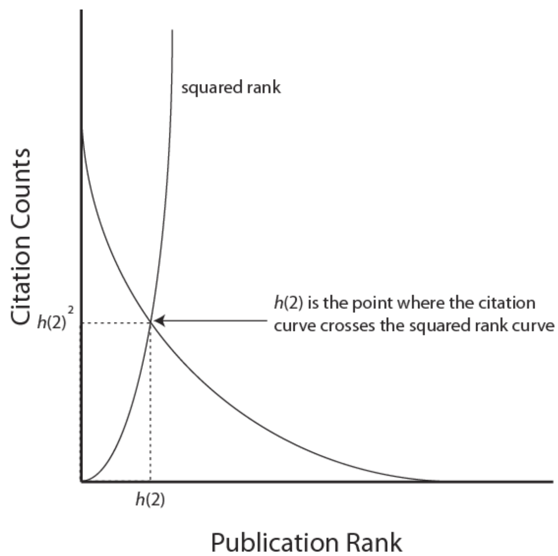

Figure 6. Graphical representation of the h(2) index.

This index was proposed in part to ease the verification of authorship when determining a metric for authors with common or ambiguous names by reducing the number of publications which would have to be considered (when browsing papers in a database, one can ignore publications below a specified threshold of citations; $h(2)$ has a substantially higher threshold than h). Clearly, $h(2) \le h$, and publications within the $h(2)$ core are much more impactful (based on citation count) on average than papers within the h core.

Another index designed to only include highly impactful papers is **Wu's *w*-index** (Wu 2010). This index is similar to the others, but requires that papers have at least 10w citations to be included in the core. Thus an author has an index of w if w of their papers have at least 10w citations, or

$$w = \max_i \left(i \le 10C_i \right).$$

Graphically, this is identical to the *h*-index, except we are looking for the point where the citation curve crosses a line with slope equal to ten rather than one. Wu's *w*-index is always stricter in its requirements than the *h*-index (requiring 10 times as many citations for all papers at every increment); it has more strict requirements than the *h*(2)-index up to values of 10 (where both require 10 publications with at least 100 citations each), then becomes less strict than *h*(2) in its requirements as the indices move above 10. A slope of 10 is essentially an arbitrary measure and it should be obvious that there are any number of curves (both more conservative and more liberal than that used for *h*) which could be used to define the core.

The ***hg*-index** (Alonso *et al.* 2010) does not formally define a new core, but is rather an aggregate index which tries to keep the advantages of both the *h*- and *g*-indices while minimizing their disadvantages. The index is simply the geometric mean of *h* and *g*, or

$$hg = \sqrt{h \times g}.$$

It can easily be shown that $h \leq hg \leq g$. As with the other metrics in this section, the *hg*-index is measuring numbers of publications.

Table 3 shows all of the core-defining indices calculated for the sample data. The units for all of these are number of publications, so the values are directly comparable. All of these indices are also defined in such a way that they can never decrease, only increasing or staying constant across time.

Table 3. Core defining indices for each year.

Date	1997	1998	1999	2000	2001	2002	2003	2004
h-index	1	2	3	5	6	7	9	11
g-index	1	3	6	8	11	15	19	24
Tol *f*-index	1	3	4	6	7	9	12	16
Tol *t*-index	1	3	4	6	8	11	14	18
μ-index	1	3	4	7	9	10	11	17
Woeginger *w*-index	1	4	5	8	9	13	16	22
h(2)-index	1	1	2	2	3	4	4	5
Wu *w*-index	0	0	1	1	2	2	3	3
hg-index	1.00	2.45	4.24	6.32	8.12	10.25	13.08	16.25

Date	2005	2006	2007	2008	2009	2010	2011
h-index	13	15	17	19	23	24	24
g-index	29	32	35	35	38	44	48
Tol *f*-index	18	21	24	27	30	32	34
Tol *t*-index	21	24	28	31	33	36	38
μ-index	19	21	25	29	31	33	35
Woeginger *w*-index	24	29	32	34	35	39	41
h(2)-index	6	7	7	8	8	8	9
Wu *w*-index	4	6	7	7	7	8	8
hg-index	19.42	21.91	24.39	25.79	29.56	32.50	33.94

Although the defined cores may be substantially larger or smaller than *h*, most of these indices appear to increase at approximately the same rate (e.g., most of the indices roughly doubled between 2004 and 2011). It is interesting that the *hg*-index (the geometric mean of *h* and *g*) is very similar at every time point to Tol's *f*-index (which is based on the harmonic mean of citations within the core).

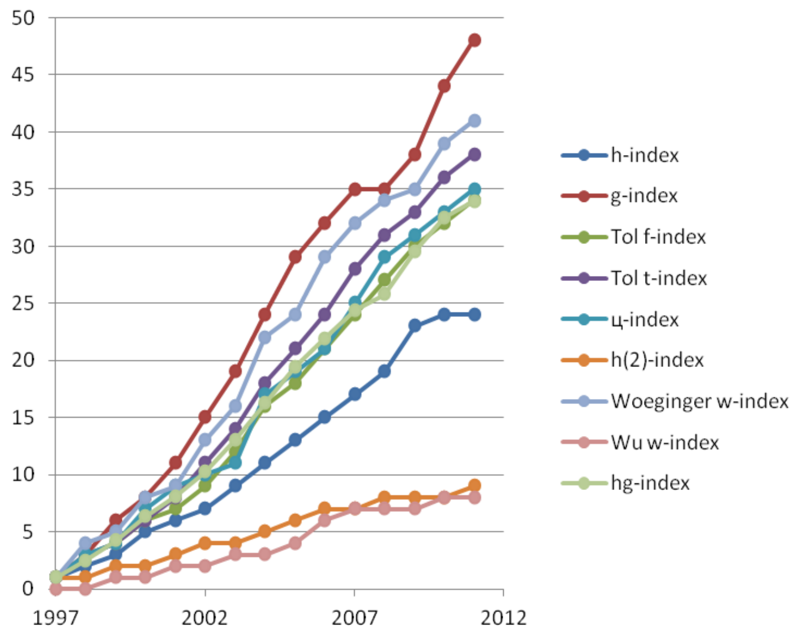

Figure 7. Change in some of the core measuring impact factors through time.

It should be abundantly clear that there are two independent aspects to defining the impact core. The first is a ranked citation curve, which represents some aspect of all citations to that rank, such as the minimum (as for *h*) or arithmetic mean (as for *g*). The second is a threshold function, usually taken to be a straight line with slope one ($y = x$) as in *h* and *g*, but which can also have a different slope (Wu's w-index uses $y = 10x$) or represent a non-linear function (*h*(2) uses $y = x^2$). Clearly other novel indices combining different aspects of these could easily be constructed (and probably justified by someone). For example, a ***g*(2)-index** (invented here as far as I know) would combine the arithmetic mean of the top citations with the $y = x^2$ threshold, thus defining *g*(2) as the largest value for which *g*(2) publications have at least an average of $g(2)^2$ citations. My *g*(2)-index for the 2011 data would be 13, meaning my top 13 most cited publications average at least 169 citations, while my top 14 most cited publications average fewer than 196 citations. Not surprisingly, this new index falls between *g* and *h*(2) since it combines the looser criteria for defining the ranked publication curve (average rather than minimum) with the stricter definition of the threshold curve ($y = x^2$ rather than $y = x$). Whether this (or any other combination of functions) is at all useful or desirable is highly questionable and most experts have tended to stick with *h* or *g* as the basis of their impact factor.

Indices which Give Credit for Excess Citations

One simple disadvantage of the *h*-index is that it is restricted to integer values and only increases in steps. The **rational *h*-index** (Ruane and Tol 2008) (h^{Δ} or h_{rat}) is a continuous version of *h* which not only measures the standard *h*-index but includes the fractional progress toward the next higher value of *h*. It is *h* plus the number of additional citations necessary to reach *h* + 1. It is calculated as

$$h^{\Delta} = h + 1 - \frac{n}{2h+1}$$

where *n* is the number of citations necessary to reach the next value of *h*. The divisor, 2*h* + 1, is the maximum number of possible citations needed to move from *h* to *h* + 1 (1 additional citation for each of the *h* publications in the core plus *h* + 1 citations for a publication outside of the core with no citations). Practically speaking, *n* is the number of papers in the core with exactly *h* citations (thus needing one more to allow a move to *h* + 1) plus $h + 1 - C_{h+1}$ (the number of citations the $h + 1^{\text{th}}$ ranked publication needs to reach *h* + 1 citations).

In a similar manner, one can calculate the **real *h*-index** (Guns and Rousseau 2009) as the point at which the linear interpolation between *h* and *h* + 1 crosses the line with slope one,

$$h_r = \frac{(h+1)C_h - hC_{h+1}}{1 - C_{h+1} + C_h}.$$

The real *h*-index has the same graphical definition as *h*, except it is not restricted to the integer values and thus represents the actual point where the citation and threshold curves cross. Rational and real versions of the *g*-index have also been defined (Guns and Rousseau 2009; Tol 2008).

When proposing his *w*-index, Wu (2010) also suggests a secondary measure, ***w*(*q*)-index** where *q* is the minimal number of additional citations necessary to improve from a score of *w* to *w* + 1. It is conceptually nearly identical to the rational *h*-index except that (1) it describes the scores needed to change *w* and not *h*, (2) it is left as an integer rather than scaled to the proportion of maximum possible citations which could be needed, and (3) Wu suggests it can be calculated to not only determine distance to *w* + 1, but also to *w* + 2, *w* + 3, etc.

The ***Hj*-indices** (Dorta-González and Dorta-González 2010) are essentially a multivariate cross between h^{Δ} and *w*(*q*). Like h^{Δ}, they attempt to discriminate amongst researchers with identical *h* values by comparing the upper and lower parts of the core to measure how close an author is to moving from one *h* to a larger *h*. These indices are repeated for a series of *j*'s, where each *j* indicates the next higher value of *h* or *h* + *j*. Like *w*(*q*), the measures are in raw numbers of publications (rather than scaled) so can be a bit more cumbersome to interpret; furthermore they don't measure the missing number of citations, but rather total numbers and can include citations above-and-beyond those necessary to reach a particular *j*. The basic calculation starts with the number of papers in the central core, thus

$$H_o = h^2$$

Each subsequent value is then calculated as

$$H_j = H_{j-1} + \left(C_{h-j} - C_{h-j+1}\right)\left(h-j\right) + C_{h+j}.$$

For H_1, this is essentially the number of citations necessary to reach *h*, plus the citations currently in the next paper outside of the core, plus the minimum number of citations over *h* common to all publications within the core. It is this last part that can make interpretation so difficult since a well over-cited core can lead to very large increases in subsequent values of H_j. For example, at the beginning of 2004, my H_j indices (H_0 to H_{10}) were: 121, 162, 191, 248, 257, 288, 305, 319, 387, 407, and 568, respectively. By definition H_0 is simply the square of *h* (=11). To reach an *h* of 12 requires a minimum of $12^2 = 144$ citations. But H_1 is 162 indicating an excess of core citations beyond *h* + 1, without clearly identifying how close I am to actually reaching *h* = 12 in the manner of the rational *h*-index or an approach more similar to *w*(*q*). When comparing researchers with identical *h*-indices, however, Dorta-González and Dorta-González (2010) claim that larger values of H_2 and H_3 may be strong predictors of potential future growth in *h*.

While the rational *h*-index gives a fractional value to those citations necessary to reach the next value of *h*, the **tapered *h*-index** (Anderson *et al.* 2008) is designed to give every citation for every publication some fractional value. The best way to understand this index is to first consider the contribution of every citation to the *h*-index. To have an *h*-index of 1, an author needs a single paper with a single citation. That citation has a weight (or score) of 1, because it accounts for the entire *h* value of 1. To move to an *h*-index of 2, the author needs three additional citations: one additional citation for the original publication and two citations for a second publication. As *h* has increased by one, each of these three citations is contributing a weight (or score) of 1/3 to the total *h*-index. This is most easily illustrated by a Ferrers graph of ranked publications versus citations which shows the specific contribution of every citation to a specific value of *h*:

Ranked publication \ Citation	1	2	3	4	5	6	→
1	1	1/3	1/5	1/7	1/9	1/11	
2	1/3	1/3	1/5	1/7			
3	1/5	1/5	1/5	1/7			
4	1/7	1/7					
5	1/9						
↓							

Figure 8. Scoring for the tapered h-index.

The largest filled-in square in the upper left corner (the Durfee square) has a length equal to *h*; the contents of the square also sum to *h*. Using this logic, one can determine the credit each citation would give to a larger value of *h*, regardless of whether that *h* has been reached. Consider this graph with respect to the rational *h*-index. In the above example, *h* is 3. If one just considers the citations necessary to reach an *h* of 4, we can see that 5 of the 7 necessary citations are already present. Each of these has a weight of 1/7 (since 7 total citations are necessary); adding these to *h* we get the rational *h*-index, $h^{\Delta} = 3.71$. The tapered *h*-index is simply taking this same concept but expanding it to include all citations for all publications.

The tapered *h*-index for a specific publication is the sum of all of its scores and the total score of the index is the sum across all publications. In simple formulaic terms, the score $h_{T(i)}$ for the i^{th} ranked publication is calculated as

$$h_{T(i)} = \begin{cases} \dfrac{C_i}{2i-1} & \text{if} \quad C_i \leq i \\[2ex] \dfrac{i}{2i-1} + \displaystyle\sum_{j=i+1}^{C_i} \dfrac{1}{2j-1} & \text{if} \quad C_i > i \end{cases},$$

and the total tapered h-index is the sum of these scores for all publications,

$$h_T = \sum_{i=1}^{P} h_{T(i)}.$$

This index is consistent with the concept of the h-index, while also giving every citation some small influence on the score. It is obvious that $h \leq h_T$. If one has a few very highly cited papers in the core (papers cited well beyond h), then h_T may be substantially larger than h and can even exceed P. With a single publication, the maximum value of h is one (as long as it has a single citation). The tapered h-index can continue to grow as long as the publication is cited, however. With 8 citations, a single publication will have $h_T = 2$; 57 citations leads to $h_T = 3$; and 419 citations are required to reach $h_T = 4$. This illustrates that the effect of additional citations on a single publication is relatively small since it takes increasingly large numbers of citations within a single publication to increase the value of the index by a full step.

The ***j*-index** (Todeschini 2011) is another modification of the h-index which allows for over-cited publications in the core to increase the overall value of the index. It uses a set of fixed categorical increases over h:

k	1	2	3	4	5	6	7	8	9	10	11	12
Δh_k	500	250	100	50	25	10	5	4	3	2	1.5	1.25

$$j = h + \frac{\sum_{k=1}^{12} w_k X_k \left(h \times \Delta h_k\right)}{\sum_{k=1}^{12} w_k},$$

where w_k, the weight given to each category, is simply $1/k$, and $X_k(h \times \Delta h_k)$ is the count of publications whose citations are at least equal to $h \times \Delta h_k$. Essentially this metric adds additional scores to *h* for publications which are cited well more than that necessary for the core, with larger weight given to those much higher than the core value (500 times the core get a weight of 1, 250 times the core get a weight of 0.5, etc.).

Wohlin's *w*-index (Wohlin 2009) is similar to others that try to address the issue where that not all citations are included in the *h*-index and that many different distributions of citations can have identical *h*-indices. Unlike the other indices, however, it does not start with *h*, and instead uses a somewhat complicated procedure of dividing papers into classes based on the number of citations. Rather than give publications more weight for every citation, this index give more weight to citations as the publication moves from one class to the next. Publications with fewer than five citations are ignored (given a weight of 0). The first class represents publications with 5-9 citations; each subsequent class has a width double that of the previous class, thus the 2^{nd} class represents 10-19 citations, the 3^{rd} class 20-39 citations, etc. This structure was chosen (other classification schemes could be substituted) because citations curves are usually skewed with many publications with relatively smaller numbers of citations, and few publications with relative large numbers of citations. To calculate the metric, for each of the c' classes, one can count the number of publications within the c^{th} class, X_c. Skewed distributions are often normalized using a logarithmic transform. Therefore, one calculates the natural logarithm of the lower limit of each class as $T_c = \ln(B_c)$ where B_c is the lower limit of the c^{th} class. One can also calculate V_c as the cumulative sum of T_c for all classes from 1 to c. The *w*-index is then calculated as

$$w = \sum_{c=1}^{c'} X_c V_c$$

The *w*-index increases as a publication moves from one class to the next. Moving between larger classes gives more weight than moving between smaller classes. Because it considers citations more broadly, the *w*-index is more fine-grained than the *h*-index.

Table 4. Indices which use citations beyond the core minimum.

Date	1997	1998	1999	2000	2001	2002	2003	2004
h-index	1	2	3	5	6	7	9	11
rational *h*-index	1.33	2.80	3.71	5.73	6.69	7.87	9.89	11.96
real *h*-index	1.00	2.50	3.33	5.00	6.33	7.50	9.00	11.50
tapered *h*-index	1.33	3.74	4.86	7.23	9.46	12.13	14.87	18.35
j-index	1.00	2.18	3.33	5.41	6.65	7.79	9.77	12.16
Wu's *w*(*q*)-index	0(9)	0(4)	1(13)	1(10)	2(16)	2(10)	3(21)	3(9)
Wohlin's *w*-index	0.00	1.61	5.52	15.65	28.55	46.74	64.70	105.00

Date	2005	2006	2007	2008	2009	2010	2011
h-index	13	15	17	19	23	24	24
rational *h*-index	13.85	15.90	17.94	19.97	23.89	24.90	24.98
real *h*-index	13.00	15.00	17.50	19.50	23.00	24.00	24.50
tapered *h*-index	21.01	23.49	26.55	28.24	30.97	34.09	36.67
j-index	14.38	16.81	18.91	21.19	25.16	26.41	26.62
Wu's *w*(*q*)-index	4(5)	6(6)	7(30)	7(10)	7(1)	8(23)	8(4)
Wohlin's *w*-index	140.48	180.57	224.81	251.30	297.87	332.43	375.06

The rational and real *h*-indices both range from *h* to *h* + 1, but measure slightly different things. The primary advantage of the rational *h*-index is that it shows progress toward the next step in *h* and may increase across time steps when *h* does not. Because my upper core is well over-cited relative to *h*, the rational *h* tends to be almost one full point about *h*. The real *h*-index more accurately captures the graphical concept of the intercept between the citation curve and the line of slope one (or fitting the largest square under the curve) since the intersection point may be between integer values. The tapered *h*-index, in contrast, illustrates the effect of including all citations quite strongly since it increases by over two and a half points between 2010 and 2011 when *h* does not change at all. Although starting from *h* and seemingly on a similar scale as the previous metrics, it is not clear the *j*-index can be interpreted in as straight-forward a manner as the rational *h* or tapered *h*, reducing its potential usefulness. The fractional approach used in the rational *h* seems more useful than the integer approach used with *w*(*q*) since the value of *q* is only meaningful when comparing identical Wu *w*'s (as can be seen in consecutive years where *w* does not change, e.g., 2007 through 2009). While the other metrics are all measured in numbers of publications, making interpretation relatively straightforward, Wohlin's *w*-index is very difficult to compare to the other because it is measured in a completely different scale without strict underlying meaning.

Although largely applied to *h*, many of these approaches could be easily extended to any of the core-defining indices already described.

Indices for Describing the Core and Tail

In order to distinguish amongst researchers with identical *h*- (or other) indices, numerous metrics have been described which measure properties of the core or citation distribution beyond the core. These are largely not meant to serve as independent indices but rather as supplements to the core measure. Most of these measures are based on a core defined by *h*, but there is no reason most of them could not be applied to cores based on other indices.

Indices which Measure the Core

The simplest of these measures is to normalize the *h*-index for the total number of publications, either as a fraction or a percentage. The **normalized *h*-index** (Sidiropoulos *et al.* 2007) is

$$h^n = h/P$$

while the ***v*-index** (Riikonen and Vihinen 2008) (see below for other *v*-indices) is the same value expressed as a percentage, $v = 100h^n$. These both indicate what proportion of the publication output is contained within the core.

Table 5. The percentage of publications contained within the core.

Date	1997	1998	1999	2000	2001	2002	2003	2004
Total Publications	5	6	7	11	15	19	23	26
h-index	1	2	3	5	6	7	9	11
v-index	20.00	33.33	42.86	45.45	40.00	36.84	39.13	42.31
Date	2005	2006	2007	2008	2009	2010	2011	
Total Publications	31	32	35	35	38	44	48	
h-index	13	15	17	19	23	24	24	
v-index	41.94	46.88	48.57	54.29	60.53	54.55	50.00	

Unlike the indices which define the core or include all citations, core descriptors can decrease through time. The *v*-index fluctuates year per year, higher some years, lower others. This highlights a problem with a number of metrics, particular those which use the number of publications in a denominator: publishing new papers can decrease your score, a generally undesirable property of measures meant to record impact.

The ***a*-index** (Jin 2006; Rousseau 2006) is used to describe the citations within the core itself, being simply the average number of citations per core publication, or

$$a = N_h / h$$

The minimum value of *a* is *h* (since every one of the *h* papers must have at least *h* citations). The ***m*-index** (Bornmann *et al.* 2008a) is similar, but uses the median number of citations per core publication, rather than the mean. Because the citation distribution within the core will generally be highly skewed, the *m*-index should be a better measure of central tendency than the *a*-index.

The ***r*-index** (Jin *et al.* 2007) is a measure of the quality of the Hirsch core, designed to avoid punishing scientists with larger cores. As a simple arithmetic average, the *a*-index has the size of the core in the divisor and therefore can lead to smaller values for scientists with much larger cores than those with much smaller cores (this is not an issue of the indices are only being used to compare those with similar sized cores). The *r*-index uses the square-root of the citations in the core rather than average,

$$r = \sqrt{N_h}$$

(also note that $r = \sqrt{a \times h}$). As with *a* and *m*, the minimum value of *r* is *h*. The **r_m-index** (Panaretos and Malesios 2009) is a simple modification of the *r*-index, where one sums the square-root of the citations within the core rather than the total count:

$$r_m = \sqrt{\sum_{i=1}^{h} \sqrt{C_i}}$$

Similar to the *r*-index, the **weighted *h*-index** (Egghe and Rousseau 2008) is designed to give more weight to publications within the core as they gain citations. The primary difference is that for this metric the core is defined differently. Publications are still ranked by citation count, but instead of using the raw rank, one uses a weighted rank of

$$r_w(i) = \frac{N_i}{h} = \frac{\sum_{j=1}^{i} C_j}{h},$$

that is, the weighted rank of the i^{th} publication is the cumulative sum of citations for the top i publications, divided by the standard h-index. With these weighted ranks, one finds the last publication in the weighted core, r_0, as the largest value of i where $r_w(i) \leq C_i$ (the last publication for which the weighted rank of that publication is less than or equal to the number of citations for that publication).

$$r_o = \max_i \left(r_w(i) \leq C_i \right)$$

The weighted index is then calculated as

$$h_w = \sqrt{\sum_{i=1}^{r_0} C_i} \; ,$$

the square-root of the sum of citations for the weighted core.

Table 6. Core description indices.

Date	1997	1998	1999	2000	2001	2002	2003	2004
a-index	0.20	1.50	4.43	5.27	8.00	10.79	13.57	19.50
m-index	1.00	4.50	8.00	8.00	12.50	16.00	15.00	27.00
r-index	1.00	3.00	5.57	7.62	10.95	14.32	17.66	22.52
r_m-index	1.00	2.04	3.03	4.03	5.04	5.89	6.84	8.18
weighted *h*-index	1.00	2.45	4.36	6.08	8.83	11.66	15.17	18.17

Date	2005	2006	2007	2008	2009	2010	2011
a-index	24.13	32.75	38.80	49.31	57.42	58.77	61.04
m-index	44.00	48.00	41.00	44.00	51.00	57.00	64.50
r-index	27.35	32.37	36.85	41.55	46.71	50.85	54.13
r_m-index	9.38	10.56	11.57	12.62	13.96	14.68	15.13
weighted *h*-index	23.17	29.15	32.77	37.03	40.36	43.65	46.48

The *a*-index, *m*-index, *r*-index, r_m-index, and weighted *h*-index all represent, in some manner, the number of citations in the core. Both the *a*-index and *m*-index have the potential to decrease through time since they are based on averages. For my data, in most years both of these measures have tended to increase, showing a consistent increase in citations for all papers within the core, even as the core itself has gotten larger. Being based solely on citation counts, the *r*-, r_m-, and h_w-indices can only increase through time, although their interpretation is less straightforward than the previous indices. The simplest interpretation of *r* is that it represents the largest value of *h* which could be obtained for the citations within the core if they were distributed completely evenly among *r* publications. It is not clear that r_m has any logical interpretation but instead just serves to help distinguish researchers within identical *h*. As shown, the weighted *h*-index (which is perhaps misnamed since it is functionally more like a weighted *r*-index) is very similar to *r*, differing in value because of the difference in the definition of the core.

The **π-index** (Vinkler 2009) is similar to other measures of the quality of the Hirsch core, except that it uses its own unique definition of the core. For this index, the core publication set is defined as the top *x* publications where *x* is the square-root of the total number of citations, *i.e.*, $x = \sqrt{N_P}$, truncated down to the nearest integer (e.g., for 80 publications, the square-root of 80 is 8.944, so *x* would equal 8). The π-index is 1/100th of the total citations within this core,

$$\pi = \frac{N_x}{100}.$$

The ***q*²-index** (Cabrerizo *et al.* 2010) is another metric designed to describe the Hirsch core. It is the geometric mean of both a quantitative (*h*-index) and a qualitative (*m*-index) measure of the core,

$$q^2 = \sqrt{h \times m}$$

Its units are somewhat odd which makes direct interpretation more difficult than some other indices, but it does have the effect of evening out some of the differences between individuals with a lot of citations in a few core papers versus individuals with fewer citations in more core papers.

The ***e*-index** (Zhang 2009) is simply a measure of the excess citations in the Hirsch core beyond those necessary to produce the core itself. It is measured as

$$e = \sqrt{N_h - h^2}\ .$$

The *e*-index is the square-root of the count of citations in the upper-section of the citation graph (Figure 9).

Strictly speaking, the **maxprod** index (Kosmulski 2007) is not a measurement of the core, but seems to fit best with these indices. It is simply the maximum value for the product between the number of citations for a publication and its rank, or

$$mp = \max\left(i \times C_i\right).$$

Although, Maxprod $\geq h^2$, it will often be fairly close to h^2; when it is not, it indicates a researcher with an unusual citation distribution.

Table 7. Additional core distribution indices.

Date	1997	1998	1999	2000	2001	2002	2003	2004
π-index	0.02	0.09	0.27	0.45	0.92	1.72	2.56	3.90
q^2-index	1.00	3.00	4.90	6.32	8.66	10.58	11.62	17.23
e-index	0.00	2.24	4.69	5.74	9.17	12.49	15.20	19.65
Maxprod	2	6	19	32	72	94	142	218

Date	2005	2006	2007	2008	2009	2010	2011
π-index	5.37	7.15	9.16	11.22	14.66	17.24	19.45
q^2-index	23.92	26.83	26.40	28.91	34.25	36.99	39.34
e-index	24.06	28.69	32.70	36.95	40.66	44.83	48.52
Maxprod	308	448	532	615	750	884	1021

Each of these indices is somewhat unique and not directly comparable to each other. As shown above, being based in part on the *m*-index allows the q^2-index to decrease through time. The π-, *e*-, and maxprod-indices can only increase.

Indices which Compare the Core and Tail

As already discussed, the *h*-index divides the citation curve into three sections (Figure 9): the h^2 citations necessary to produce the score of *h* (the gray box), the extra citations in the core above-and-beyond those necessary to produce the score of *h* (those above the box or e^2), and the citations for all publications outside of the core (those in the tail to the right of the box).

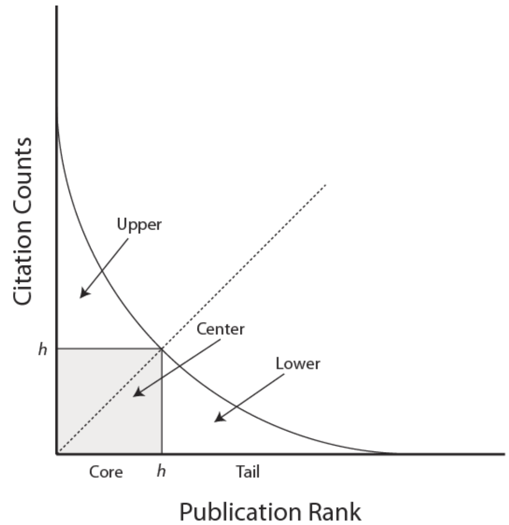

Figure 9. Citation curve with upper, center, and lower sections defined.

These can be referred to as the center (the square), upper (excess), and lower (tail) parts. The citations in the upper and center are the total citations in the core (N_h). Bornmann *et al.* (2010) suggest capturing the relative distributions of these parts by calculating the percent of total citations represented by each area. Thus

$$h^2_{upper} = \frac{N_h - h^2}{N_P} \times 100 = \frac{e^2}{N_P} \times 100$$

$$h^2_{center} = \frac{h^2}{N_P} \times 100$$

$$h^2_{lower} = \frac{N_P - N_h}{N_P} \times 100$$

Scientists with high values for the upper part and small values for the lower part are sometimes referred to as *perfectionists* (they do not publish much, but what they publish is highly impactful). Those with low values in the upper part and high values in the lower part are *mass* *producers* (lots of publications of relatively low impact). Those in the middle are *prolific* (producing an abundance of impactful papers).

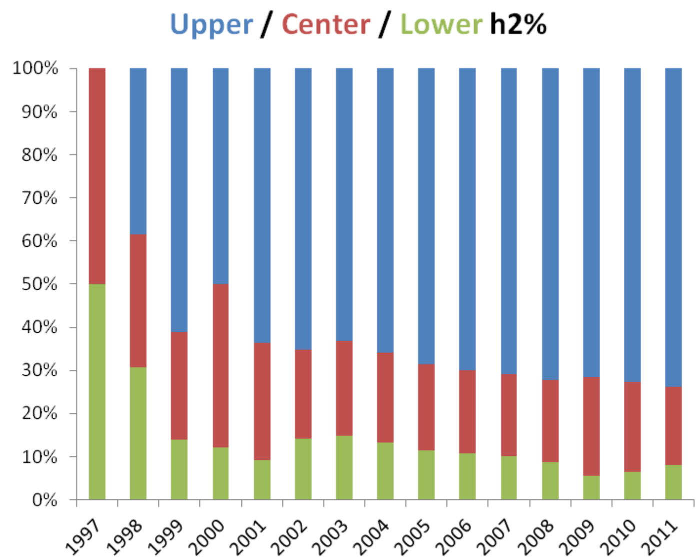

Figure 10. Relative proportions of citations in the upper (blue), center (red), and lower (green) sections of the citation curve through time.

After the first five or six years of publishing, my citations have shown a fairly stable distribution, with 7-15% of my citations occurring in the tail of the distribution, 21-30% of my citations in the center (core square) of the distribution, and 60-70% of my citations in the upper/excess part of the core. For the most part the distribution appears to have been fairly stable over most of my career, although the upper part of the core seems to have gradually increased over the last decade.

The ***k*-index** (Ye and Rousseau 2010) is a measure of the relative impact of citations within the Hirsch core to those in the tail. Specifically, it is the ratio of impact over the tail-core ratio and is calculated as

$$k = \frac{N_P N_h}{P\left(N_P - N_h\right)}.$$

This metric is specifically meant to be used in a time-series analysis where *k* is calculated for multiple time points. The usefulness of this metric is not at all clear.

Also called the **Mock *h_m*-index** (Prathap 2010b), the ***p*-index** (Prathap 2010a) is derived from mathematical modeling of the relationship of increasing numbers of publications and citations. It is a function of the total number of citations and the average citations per paper,

$$p = \sqrt[3]{N_P^2 / P}$$

The *p*-index can be considered a predictor of *h*. The ratio between *p* and *h* (the ***ph*-ratio** = *p* / *h*) reflects the sensitivity of the value to the proportion of citations in the upper core and the lower tail.

Table 8. Indices which include the tail distribution.

Date	1997	1998	1999	2000	2001	2002	2003	2004
h-index	1	2	3	5	6	7	9	11
k-index	0.40	4.88	31.89	43.50	88.00	75.84	91.94	146.25
p-index	0.93	3.04	5.70	7.34	10.51	14.43	17.99	23.61
ph-ratio	0.93	1.52	1.90	1.47	1.75	2.06	2.00	2.15
Date	2005	2006	2007	2008	2009	2010	2011	
h-index	13	15	17	19	23	24	24	
k-index	210.20	300.89	387.74	568.32	1013.85	893.86	748.93	
p-index	28.45	35.09	40.22	46.73	52.02	55.84	59.63	
ph-ratio	2.19	2.34	2.37	2.46	2.26	2.33	2.48	

None of these indices seems to be particularly useful: the interpretation of the *k*-index is unclear while the *p*-index is a predictor of the *h*-index if one assumes that citations increase under a specific mathematical model.

The **multidimensional *h*-index** (García-Pérez 2009) is a simple expansion of the original *h*-index used to separate amongst individuals with identical *h*-indices. The concept is to calculate the *h*-index from all *P* publications (this would be the first *h*-index or h_1). One can then calculate a second *h*-index, h_2, from the $P - h_1$ remaining publications. Graphically, this is finding the largest square which can fit in the tail to the right of the original square represented by h_1 (Figure 11). A third, h_3, can be calculated from the $P - h_1 - h_2$ remaining publications, etc., continuing until one reaches publications with 0 citations. It should be obvious that $h_1 \geq h_2 \geq h_3 \ldots$

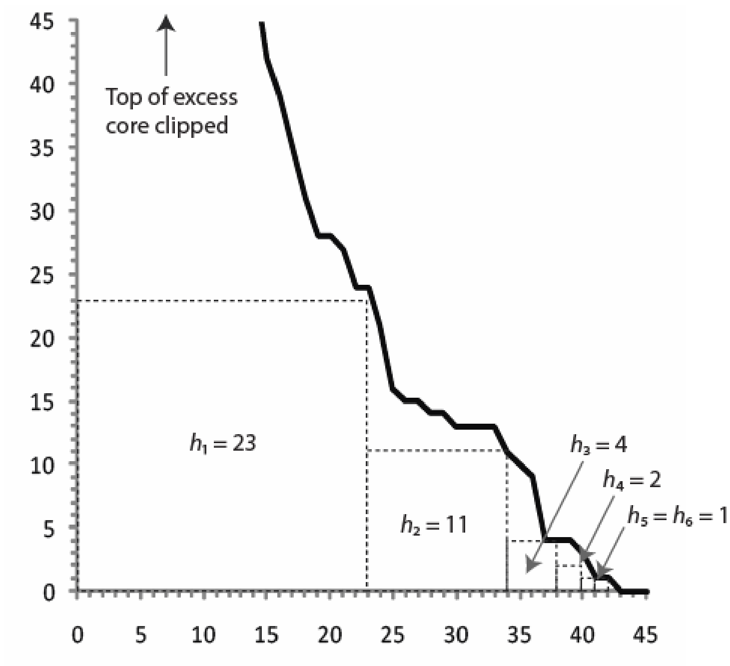

Figure 11. Graphical illustration of multidimensional h-index for 2011 data. The standard h (=h_1) is 23. The largest square which can fit in the remaining tail is of size 11 (h_2). This is followed by squares of 4, 2, 1, and 1.

Unlike most of the other indices, this index set is primarily focused on the tail of the distribution, ignoring the excess/upper part of the citation curve completely. Because it is simply recalculating

h for a smaller data set, its interpretation is quite straightforward and certainly could serve as a solid method of distinguishing individuals with identical h.

Indices which Account for Multiple Authors

Whether impact factors should be adjusted for multiple authors or authorship role is controversial, but is also not a unique issue to impact factor determination, since similar arguments are often made when looking at publication counts for promotion and tenure or job hiring decisions.

One approach is to calculate a **major contribution h-index**, h_{maj}, only including those papers to which the author has made a major contribution (Hu *et al.* 2010). This metric is otherwise determined just like the *h*-index. How one defines “major contribution” is clearly debatable.

The simplest correction for multiple authors is the $\boldsymbol{h_i}$**-index** (Batista *et al.* 2006). This index is simply the *h*-index divided by the average number of authors in the core publications, or

$$h_i = \frac{h^2}{\sum_{i=1}^{h} A_i}$$

If every publication in the core is solo-authored than $h_i = h$. This can be an extremely harsh correction. A single core publication with a large number of co-authors may skew the average and thus lower ones impact factor tremendously. Use of the median rather than the mean might be a fairer approach.

The **pure *h*-index** (Wan *et al.* 2007) is similar to the h_i -index in that it attempts to adjust for multiple authors. The index allows for different methods of assigning authorship credit, but without information on authorship order, the only way to calculate it is to assume equal fractional credit per author, which essentially means the metric is simply the *h*-index divided by the square-root of the average number of authors in the core, thus differing from h_i only by the square-root in the denominator (which makes the pure *h*-index less harsh than h_i by not punishing co-authorship as severely).

$$h_p = \frac{h}{\sqrt{\frac{\sum_{i=1}^{h} A_i}{h}}}.$$

With authorship order (or direct information on authorship credit per paper), a number of different authorship credit schemes could be implemented which changes the association between h_i and h_p (Abbas 2011; Wan *et al.* 2007). Assuming authorship order directly correlates with effort, proportional (or arithmetic) assignment of author credit for each publication would be

$$E_i = \frac{A_i\left(A_i + 1\right)}{2\left(A_i + 1 - A_i'\right)},$$

where A_i' is the position of the author within the full author list (*i.e.*, an integer from 1 to A_i). For geometric assignment of author credit

$$E_i = \frac{2^{A_i} - 1}{2^{\left(A_i - A_i'\right)}}.$$

In both cases, the pure *h*-index then becomes

$$h_p = \frac{h}{\sqrt{\frac{\sum_{i=1}^{h} E_i}{h}}}.$$

These schemes make assumptions about author order which are not true for all fields and which can easily be violated for any number of legitimate reasons (e.g., equal credit among some, but not all authors).

The **adapted pure *h*-index** (Chai *et al.* 2008) uses very similar logic to the pure *h*-index, except that it estimates its own core rather than simply relying on the standard Hirsch core. Each paper has an effective citation count calculated as the number of citations divided by the square-root of the equivalent number of authors (as for the pure *h*-index, for these purposes we are assuming that every author gets equal credit since we do not have information to indicate otherwise, therefore the equivalent number of authors is equal to the number of authors), $C_i^* = C_i / \sqrt{A_i}$. Publications are ranked according to these new citation values and the *h*-equivalent value, h_e, is found as the largest rank for which the rank is less than the number of equivalent citations, or

$h_e = \max_i \left(i \leq C_i^* \right)$. The adapted pure *h*-index is calculated by interpolating between this value and the next largest, as

$$h_{ap} = \frac{(h_e + 1) C_{h_e}^* - h_e C_{h_e+1}^*}{C_{h_e}^* - C_{h_e+1}^* + 1}.$$

Just as with the pure *h*-index, the adapted pure *h*-index can also be used with proportional and geometric authorship credit based on authorship order. In these cases $C_i^* = C_i / \sqrt{E_i}$, with E_i calculated as above and all other equations identical

Just like the adapted pure *h*-index, the **normalized h_i-index** (Wohlin 2009) is designed to adjust the *h*-index for multiple authors by adjusting the citation count by the number of authors. The primary difference is the new citation value is calculated by dividing by the number of authors (C_i / A_i) rather than the square-root of the number of authors. Publications are again ranked by these new citation per author values and the normalized h_i-index is calculated in the same manner as the *h*-index, that is an author has a normalized h_i-index of $h_{i\text{-}norm}$ when $h_{i\text{-}norm}$ of their publications has at least $h_{i\text{-}norm}$ citations per author, or

$$h_{i-norm} = \max_i \left(i \leq C_i / A_i \right).$$

This is identical to what Egghe (2008) called the **fractional citation *h*-index, h_f** and was again re-invented by Abbas (2011) as the **equally-weighted *h*-index, h_e** (despite discussing both, he seems not to realize that h_e is logically identical to h_f). Egghe (2008) applied this same concept to the *g*-index to produce a **fractional citation *g*-index, g_f,** as well.

$$g_f = \max_i \left(i^2 \leq \sum_{j=1}^{i} \frac{C_i}{A_i} \right).$$

Abbas (2011) also described a **position-weighted *h*-index, h_p**, which uses the same position-based proportional assignment of authorship credit described above to weight citation counts prior to ranking. The major difference between this and the adapted pure *h*-index with proportional weighting is that the adapted pure version takes the square-root of the weight. For each publication, calculate a weighted citation count as the product of the citation count and the author-order-based weight:

$$E_i = C_i \frac{2\left(A_i + 1 - A_i'\right)}{A_i\left(A_i + 1\right)}$$

These values are then ranked for all publications, and h_p is calculated as other similar metrics:

$$h_p = \max_i \left(i \leq E_i\right).$$

Abas (2011) also describes a pair of weighted citation aggregate measures. The first is the **weighted citation aggregate, ψ**,which is just the weighted sum of all citations, calculated with either equal (fractional) or proportional weights:

$$\psi_e = \sum_{i=1}^{P} \frac{C_i}{A_i}$$

and

$$\psi_p = \sum_{i=1}^{P} C_i \frac{2\left(A_i + 1 - A_i'\right)}{A_i\left(A_i + 1\right)}.$$

The second is the **weighted citation H-cut, ξ,** simply the weighted sum of citations within the h-core, with the core defined by h_p or h_e (=$h_{i\text{-}norm}$ = h_f) for proportional and fractional weighting, respectively.

$$\xi_e = \sum_{i=1}^{h_e} \frac{C_i}{A_i}$$

and

$$\xi_p = \sum_{i=1}^{h_p} C_i \frac{2\left(A_i + 1 - A_i'\right)}{A_i\left(A_i + 1\right)}.$$

These are very to the previously discussed core metrics, except with author weighting.

Thus far, most of these indices have corrected the number of citations for the number authors before calculating the core. An alternate approach is to leave the citation counts alone but correct the publication rank using the ***h*$_m$-index** (Schreiber 2008b), also called the **fractional paper *h*** or ***h*$_F$-index** when it was independently derived by Egghe (2008). For this index, one still ranks publications in order of citation count, but rather than counting the rank of the i^{th} paper as i, one calculates the rank as the cumulative sum of $1/A_i$. Put in formal terms, for calculating the traditional h-index, the rank of the i^{th} paper is

$$r_i = \sum_{j=1}^{i} 1 \text{ or } r_i = i.$$

For this new index, the effective rank of the i^{th} paper is instead determined by

$$r_{eff}(i) = \sum_{j=1}^{i} 1/A_j$$

The h_m-index becomes the largest value of $r_{eff}(i)$ for which $r_{eff}(i) \le C_i$.

$$h_m = \max_{r_{eff}(i)}\left(r_{eff}(i) \le C_i\right)$$

The **fractional *g*-index** (Egghe 2008) is the logical extension of fractional counting of authorship in the h_m-index, but applied to the g-index. The effective ranks are calculated as above and the g_F-index is the largest value of $r_{eff}(i)$ for which $r_{eff}(i)^2 \le N_i$.

$$g_F = \max_{r_{eff}(i)} \left(r_{eff}(i)^2 \le N_i \right).$$

The ***gh*-index** (Galam 2011) is fairly similar to the above indices (particularly the normalized h_i-index/fractional citation *h*-index), adjusting both publication counts and citation counts by fractional authorship credit, but advocates a more complicated scheme for allocating credit among the A_i authors (called Tailor Based Allocation) which requires knowledge of author order as well as designation of values for two additional parameters, which makes general calculation and consistent use of this metric much more difficult. Liu and Fang (2012) propose a similarly complicated scheme for multiple author credit based on "Combined Credit Allocation" to produce modified versions of both the *h*- and *g*-indices.

Similar arguments could be applied to many of the other indices; for example, Prathap (2011) describes a **fractional *p*-index** which combines the fraction authorship credit described above with the *p*-index already discussed. In this case, both papers citation counts are divided by the number of authors, such that

$N'_P = \sum_{i=1}^{P} C_i / A_i$ and $P' = \sum_{i=1}^{P} 1 / A_i$, with the fractional *p*-index being

$$p_f = \sqrt[3]{N_P'^2 / P'}.$$

He also suggests a **harmonic *p*-index**, whether authorship credit based on order is based on a harmonic weighting. The A_i' author receives weighted credit equal to

$$r_i = \frac{\left(1 / A_i' \right)}{1 + 1/2 + \ldots + 1/A_i}$$, which leads to

$N'_H = \sum_{i=1}^{P} C_i r_i$ and $P' = \sum_{i=1}^{P} r_i$, and finally

$$p_h = \sqrt[3]{N'^2_H / P'}\,.$$

The $\bar{h}$**-index** (Hirsch 2010) takes a somewhat different approach from the other metrics. Strictly speaking, this metric includes papers in the $\bar{h}$ core if it has at least $\bar{h}$ citations and also belongs to the $\bar{h}$ core of every coauthor. The slightly looser, more practical definition, changes the last requirement to each paper belonging to the *h* core of each coauthor (rather than then $\bar{h}$ core) since that is somewhat easier to calculate. Either way, this metric is difficult to calculate because it minimally requires the *h*-index for every author, a non-trivial data collection task in some circumstances.

The profit indices (Aziz and Rozing 2013) attempt to measure the effect of collaboration on an author's impact. They use a harmonic weighting algorithm and information on author order (assuming that authors in the middle of an author list had the least impact) to estimate weights for each publication. The weight given to the i^{th} publication is

$$w_i = \frac{1 + \left| A_i + 1 - 2A'_i \right|}{\frac{1}{2} n^2 + n\left(1 - D\right)},$$

where D is 0 if A_i is even and $1/2n$ if A_i is odd. The sum of w_i for all publications is the number of "monograph equivalents" (a monograph being defined a single-authored publication). The **profit (*p*)-index** is the relative contribution of collaborators to an individual's total publication record, or

$$p = 1 - \frac{\sum_{i=1}^{P} w_i}{P}.$$

This value ranges from 0 to 1, with 0 indicating no contribution of co-authors (all solo-authored papers) and 1 meaning complete contribution from co-authors (a value of exactly 1 is impossible). To look at actual impact, the normal citation counts for each publication can be weighted by w_i, with a **profit adjusted h-index** (h_a-index) calculated in the standard manner using these weighted citation counts:

$$h_a = \max_i \left(i \le C_i w_i \right),$$

where the $C_i w_i$ products have been rank-ordered. Finally, the **profit *h*-index** (p_h) is the ratio between the adjusted value and the normal *h*-index, roughly indicating the relative contribution of collaborators to an individual's *h*-index

$$p_h = 1 - \frac{h_a}{h}.$$

The profit metrics are interesting, but make a number of assumptions about author order that may easily be violated; however, it is not clear how important these assumptions may be to the overall conclusions one may obtain from them.

Because there are so many potential methods for weighting authorship, there are more author-based variants than for any other type of adjustment.

Table 9. Indices which correct for co-authorship.

Date	1997	1998	1999	2000	2001	2002	2003	2004
h-index	1	2	3	5	6	7	9	11
h_i-index	0.33	0.80	1.13	1.67	2.25	2.72	3.52	3.10
pure *h*-index (fractional)	0.58	1.26	1.84	2.89	3.67	4.37	5.63	5.84
pure *h*-index (proportional)	0.71	1.41	1.96	3.18	4.02	4.81	6.33	6.29
pure *h*-index (geometric)	0.76	1.30	1.52	2.44	3.13	3.82	5.30	1.49
adapted pure *h*-index (fract)	0.00	2.06	2.94	4.00	5.65	6.75	8.20	10.27
adapted pure *h*-index (prop)	0.00	1.92	2.65	4.00	5.27	6.68	8.14	9.92
adapted pure *h*-index (geo)	0.00	1.93	2.60	3.82	5.10	6.61	8.14	9.87
normalized h_i-index/h_f-index	0	1	2	3	4	6	7	8
h_m-index/h_F-index	0.58	1.17	1.42	2.75	3.00	4.25	5.67	7.67
weighted *h*-index h_p (prop)	0	1	2	2	3	5	6	8
w. citation aggregate (fract)	0.58	5.42	13.75	25.00	53.42	97.68	151.58	235.76
w. citation aggregate (prop)	0.70	5.53	14.77	27.13	62.07	116.46	183.57	284.25
w. citation H-cut (fract)	0.00	2.00	9.00	17.00	39.50	76.50	119.83	184.00
w. citation H-cut (prop)	0.00	3.00	11.50	17.50	47.00	93.67	149.50	238.67
g-index	1	3	6	8	11	15	19	24
g_f-index	0	1	3	4	6	9	11	14
g_F-index	1.17	2.67	2.92	5.50	8.50	9.92	11.34	12.84
p-index	0.93	3.04	5.70	7.34	10.51	14.43	17.99	23.61
fractional *p*-index	0.54	2.22	4.02	4.84	6.95	9.87	12.65	16.30
harmonic *p*-index	0.62	2.42	4.40	5.26	7.84	11.40	14.48	18.53
profit (*p*)-index	0.56	0.55	0.57	0.49	0.43	0.47	0.49	0.49
profit adjusted *h*-index (h_a)	0	1	2	4	5	6	7	8
profit *h*-index (p_h)	1.00	0.50	0.33	0.20	0.17	0.14	0.22	0.27

Date	2005	2006	2007	2008	2009	2010	2011
h-index	13	15	17	19	23	24	24
h_i-index	4.02	4.02	4.90	5.64	7.56	7.78	7.68
pure *h*-index (fractional)	7.23	7.76	9.13	10.35	13.18	13.67	13.58
pure *h*-index (proportional)	7.79	8.67	10.19	11.56	14.57	15.20	15.13
pure *h*-index (geometric)	1.91	2.35	2.83	3.33	4.41	4.69	4.69
adapted pure *h*-index (fract)	12.33	14.00	15.71	18.00	18.89	20.39	22.57
adapted pure *h*-index (prop)	11.53	13.46	15.39	17.60	19.77	21.00	22.04
adapted pure *h*-index (geo)	11.53	13.00	14.43	17.12	19.24	20.27	21.50
normalized h_i-index/h_f-index	10	12	14	15	17	19	20
h_m-index/h_F-index	9.26	10.59	12.18	13.43	14.68	15.52	17.39
weighted *h*-index h_p (prop)	10	11	13	14	16	17	19
w. citation aggregate (fract)	339.53	490.09	636.78	802.38	993.89	1192.90	1371.95
w. citation aggregate (prop)	406.33	573.23	739.33	919.31	1126.92	1344.61	1548.77
w. citation H-cut (fract)	274.33	393.31	523.24	662.69	847.88	1052.89	1209.56
w. citation H-cut (prop)	348.83	480.60	633.63	789.99	991.64	1191.08	1389.77
g-index	29	32	35	35	38	44	48
g_f-index	17	21	24	28	31	34	36
g_F-index	17.18	17.68	19.01	19.01	21.51	23.09	24.67
p-index	28.45	35.09	40.22	46.31	51.59	55.45	59.23
fractional *p*-index	18.86	23.86	27.73	32.24	35.70	39.39	42.30
harmonic *p*-index	21.44	26.70	31.11	35.91	39.37	43.36	46.71
profit (*p*)-index	0.44	0.44	0.46	0.47	0.44	0.49	0.50
profit adjusted *h*-index (h_a)	10	12	14	15	17	17	19
profit *h*-index (p_h)	0.23	0.20	0.18	0.21	0.26	0.29	0.21

The two indices which rely exclusively on the *h*-core, h_i and pure *h*, tend to show the largest "correction" for co-authorship. The other indices all estimate their own core, which leads to a more moderate authorship correction. As one would expect, the adapted pure *h*-index leads to less of a correction than the normalized h_i-index since it divides the citation count by the square-root of the author count rather than the full author count. For my publications, in most (but not all) cases, adjusting the rank for the number of authors (h_F and g_F) seems to have a greater effect than adjusting the citation count (h_f and g_f). Furthermore, when authorship order is taken into account (assuming, incorrectly, that it fully represents credit), we find that proportional assignment of credit generally makes little difference (for me) over strict unordered fractional assignment. Geometric assignment of credit can have a larger effect.

The profit indices indicate that about half of my publications have consistently been attributable to collaborator contributions, but only about 20-25% of my *h*-index is due to collaboration. Both of these values are quite a bit lower than those reported by the original creators of these indices, perhaps indicating my work is more independent than average.

I am not particularly convinced that correcting impact metrics for co-authorship is necessary or desirable, but if one wishes to do so, it seems best to avoid the metrics which do not recalculate the core (h_i and h_p) since they seem to lead to an overcorrection. Among the other co-author-correction indices, it is not at all clear which approach may be best; as with all such indices, consistency in use and application may be more important than specific index choice. Generally speaking, these metrics can be difficult to calculate because they minimally require authorship counts and maximally require information on authorship order or credit.

Indices which Account for Self-Citations

As already discussed, the most common way to deal with self-citations is simply to remove them from the raw citation counts and then calculate any and all indices with this modified citation curve. Schreiber (2007) referred to this approach as the **sharpened *h*-index**.

The ***b*-index** (Brown 2009) is designed to correct *h* for self-citations, without actually having to check the citation records for every publication. It assumes that an author's self-citation rate is fairly consistent across publications such that, on average, a fraction *k* of the citations are from other authors. Assuming that citations follow a Zipfian distribution and that empirically derived estimates of the shape of this distribution are reasonable, one finds the index

$$b = hk^{3/4}$$

where *b* is an estimate of the *h*-index corrected for self-citations.

Below I am reporting a number of measures which reflect self-citation rates. First, I have recorded two different types of self-citations. The fist represents citations of my own work in my own papers. The second includes not only the citations from the first category, but also any citations my coauthors have made of our coauthored papers when I am not a coauthor of the citing publication. While self-citation information can generally be difficult to collect, the second category is clearly more difficult than the first.

Table 10. Self-citation measures and metrics, based only on my self-citations.

Date	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
h-index	1	2	3	5	6	7	9	11	13	15	17	19	23	24	24
total self-citation rate	1.00	0.15	0.08	0.11	0.07	0.08	0.07	0.06	0.06	0.05	0.04	0.04	0.04	0.03	0.03
average self-citation rate	0.40	0.19	0.22	0.21	0.20	0.31	0.16	0.13	0.18	0.13	0.13	0.10	0.09	0.08	0.09
sharpened *h*-index	0	2	3	5	6	7	8	10	12	15	16	18	20	22	22
b-index (based on avg rate)	0.68	1.70	2.49	4.20	5.06	5.33	7.88	9.91	11.20	13.52	15.38	17.58	21.38	22.51	22.34
b-index (assume 10% rate)	0.92	1.85	2.77	4.62	5.54	6.47	8.32	10.16	12.01	13.86	15.71	17.56	21.25	22.18	22.18

Table 10 shows the results for my own self-citations only. The total self-citation rate indicates the total proportion of all of my citations which are self-cited. The average self-citation rate is the mean rate across publications. Since some publications with very low overall citation rates may have fairly high self-citation rates (e.g., a publication with 2 out of 4 citations being self-published has a rate of 0.5) while others may have low self-citation rates (particularly when the overall citation rate is very high: my most cited publication has over 1000 citations; with 5 self citations the observed rate is only 0.005 – even if I self-cited it 50 times the rate would only be 0.05), the average rate tends to skew much higher than the total rate. The *b*-index is reported based on the actual observed average rate of self-citation as well as an assumed 10% rate.

Table 11 contains the same metrics except based on both my own and my coauthors' self-citations.

Table 11. Self-citation measures and metrics, based on both my own and my coauthors' self-citations.

Date	1997	1998	1999	2000	2001	2002	2003	2004
h-index	1	2	3	5	6	7	9	11
total self-citation rate	1.00	0.15	0.33	0.24	0.17	0.14	0.12	0.10
average self-citation rate	0.40	0.19	0.36	0.27	0.25	0.35	0.19	0.19
sharpened *h*-index	0	2	3	5	6	7	8	10
b-index (based on avg rate)	0.68	1.70	2.15	3.95	4.86	5.08	7.67	9.40
Date	2005	2006	2007	2008	2009	2010	2011	
h-index	13	15	17	19	23	24	24	
total self-citation rate	0.09	0.07	0.07	0.06	0.06	0.06	0.06	
average self-citation rate	0.21	0.15	0.14	0.12	0.12	0.14	0.15	
sharpened *h*-index	12	15	16	18	20	22	22	
b-index (based on avg rate)	10.87	13.26	15.14	17.28	20.82	21.41	21.32	

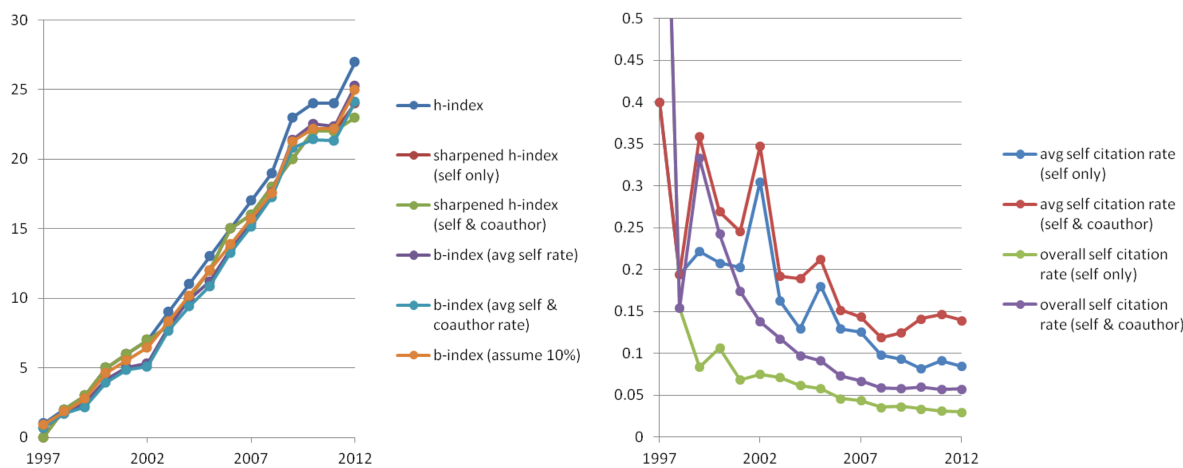

Figure 12. Self-citation effects through time. (A) Comparison of h-index and self-citation corrected indices. (B) Observed self-citation rates over time.

For the most part, self-citation has a small, but consistent effect on my *h*-index: as my *h*-index has grown through time, the sharpened *h*-index has tended to be a point or two lower. Although the observed self-citation rate when co-authors are included is about double that when they are excluded, the sharpened *h*-index appears to be unaffected by including or excluding of my co-authors' citations in the calculation. Apparently, very few of my publications are on the border of the core, and those that are tend to only need one or two additional citations to be driven into the core (in 2011, the average number of self-citations for all of my publications was only 2).

Not surprisingly, in the early part of my career, self-citations tended to make up a much larger proportion of all citations, with the rates gradually decreasing through time to something of a steady state in recent years. My average self-citation rate based on my own publications is slightly under 10% while that when co-authors are included is a bit over 10%, making a general estimate of 10% for use in the *b*-index to be reasonable. Generally, the *b*-index is very close to the sharpened *h*-index, indicating that it may be a very easy way to correct for for self-citation.

Of course, unless one has a reason to believe that some authors are self-citing at a substantially higher rate than others, then for comparison purposes, this correction is unnecessary since it will affect everyone equally. My own observed self-citation rate when coauthor citations are included appears to be very similar to estimated rates for ecologists (Leblond 2012). As the *h*-index increases in popularity, there is always a danger for manipulation through self-citation (Bartneck and Kokkelmans 2011). Generally speaking, however, self-citations tend to decrease through time and (for obvious reasons) the effect of self-citations tend to be strongest only for those with low impact factor scores (Costas *et al.* 2010; Engqvist and Frommen 2008, 2010).

Indices which Account for Time

There are a number of different reasons one might wish to correct an impact factor for time, including comparing researchers of different academic age and measuring recent impact rather than career impact. As already described above, in the original *h*-index paper (Hirsch 2005), Hirsch suggested the *m* quotient as a simple means of estimating the trajectory of *h* through time. One approach to estimate immediacy of impact is to calculate *h* (or any other metric) normally, but only including citations from a specific time interval (e.g., the last 5 years); this would measure the current impact rather than the life-time impact of a researcher (Google Scholar currently reports a both total and a 5 year *h*-index). Naturally, there have been many other suggested adaptations and corrections.

The first two described metrics are time-dependent core descriptors. The ***ar*-index** (Jin 2007; Jin *et al.* 2007) is an adaptation of the *r*-index which includes time. Rather than being the square-root of the total citations within the core, it is the square-root of the citations per year within the core:

$$ar = \sqrt{\sum_{i=1}^{h} \frac{C_i}{Y_{Now} - Y_i}}.$$

The denominator of the summation, $Y_{\mathrm{Now}} - Y_i$, is the age of each article. As an age-dependent measure, this metric can decrease through time (as opposed to many of the other measures which can only stay flat or increase), allowing one to get an estimate of current or recent productivity instead of just global/total productivity.

Similarly, the **dynamic *h*-type-index** (Rousseau and Ye 2008) is a measure of both the size of the core as well as how the core is changing through time. The basic index is

$$h_{dyn} = r \times v_h$$

where *r* is the *r*-index and v_h is a measure of the rate of change of *h* through time. Rousseau and Ye (2008) suggest a number of ways to estimate v_h, including that it be determined over a fixed time interval (e.g., 5 or 10 years) to make it more contemporary. They also suggest one use the rational *h* rather than *h* because its finer grained resolution will allow for better estimates of the rate of change of *h*. For comparison purposes, I am estimating v_h for a given date as the slope of the regression of the rational *h* against time for all data up to that date.

The ***hpd*-index** (Kosmulski 2009) is very similar to the *h*-index, except that it adjusts for the age of a publication. Rather than adjust per year, the metric is adjusted per decade. Thus if

$$cpd_i = \frac{10C_i}{Y_{Now} - Y_i}$$

is the number of citations an article has per decade, then the *hpd*-index for an author is the largest rank for which *hpd* of their publications (ranked by cpd_i rather than C_i) have $cpd \geq hpd$.

$$hpd = \max_i \left(i \leq cpd_i \right)$$

The **contemporary *h*-index** (Sidiropoulos *et al.* 2007) is designed to give more weight to the citations of recent publications and less weight to the citations of older publications. In its most general form, the contemporary score for a specific publication is

$$S_i^C = \gamma \left(Y_{Now} - Y_i + 1 \right)^{-\delta} C_i$$

The contemporary *h*-index for an author, h^C, is calculated similarly to the standard *h*-index, in that an author has a score of h^C if h^C of their articles (ranked by S^C) have $S^C \geq h^C$.

$$h^c = \max_i \left(i \leq S_i^c \right)$$

In their example, Sidiropuolos *et al*. (2007) set $\gamma = 4$ and $\delta = 1$. These choices have the consequence of making this metric virtually identical to *hpd*, except measured on a four year cycle rather than a decade. For this and similar measures (described below) the optimal time window (represented by γ in this particular index) for calculating these indices is unclear and likely varies by discipline (Wang In press).

The **trend *h*-index** (Sidiropoulos *et al*. 2007) is essentially the opposite of the contemporary *h*-index. It is designed to measure how current an author's impact is by how recently they are being cited. The trend score for a publication is measured as

$$S_i^t = \gamma \sum_{x=1}^{C_i} \left(Y_{Now} - Y_x^{C_i} + 1 \right)^{-\delta},$$

where γ and δ are parameters (often set to 4 and 1, respectively, just as with the contemporary *h*-index) and $Y_x^{C_i}$ is the *year of the x*th *citation for publication i*. The trend *h*-index is the largest value for which an author has h^t publications with at least $S^t \geq h^t$.

$$h^t = \max_i \left(i \leq S_i^t \right)$$

The problem with this index is it requires knowing the year of every citation; this requirement makes the trend *h*-index substantially more difficult to calculate than many other indices. Rons and Amez (2008, 2009) proposed a logically similar, but much more complicated measure, **Impact Vitality**, with the same problem. If C^x is the total number of citations (across all publications) from year *x*, and *w* is the number of years back from the present one wishes to calculate the metric for, then

$$IV(w) = \frac{w\left(\frac{\sum_{i=1}^{w} C^{Y_{Now}-w} / i}{\sum_{i=1}^{w} C^{Y_{Now}-w}}\right) - 1}{\left(\sum_{i=1}^{w} 1/i\right) - 1}.$$

The numerator of the numerator is the sum of citation counts divided by their age for the window of time in question; the denominator of the numerator is the total number of citations for the same window of time. An impact vitality score of 1 indicates that the number of citations is approximately constant over time. A value above 1 indicates that the number of citations is increasing through time, while a value below 1 indicates the number of citations is decreasing through time. Individuals with very different total numbers of citations can have identical scores because the metric is focused on proportional change and not absolute numbers. However, even beyond the issues of more difficult data collection, this metric has odd properties because of its overwhelming focus on immediacy. It would produce a higher score for someone with just 1 citation a year ago and no citations 2 years ago than another person with 1,000 citations 2 years ago and no citations one year ago.

The **specific-impact *s*-index** (De Visscher 2010) is designed to avoid the age-bias of other indices as well as not penalizing fields where citations may lag due to the speed of the publication process. It is designed to predict the total number of citations a set of publications will have at a time infinitely in the future, assuming exponential aging of the citation process. The *s*-index is a measure of the projected citation rate per publication (rather than the actual citation rate per publication). The practical definition is

$$s = \frac{N_P}{10\sum_{i=1}^{P} 1 - e^{-0.1\left(Y_{Now} - Y_i\right)}},$$

where s is a measure of the citation rate per publication (divided by 10) projected to time infinity. The actual prediction of the total number of citations an author would have at time infinity would therefore be $10sP$.

Franceschini and Maisano's ***f*-index** (Franceschini and Maisano 2010) is designed as a complement to the h-index. It is a measure of the time-width of impact. It is the range of time for publications with at least one citation and is calculated as

$$f = \text{Max}(Y_i \mid C_i > 0) - \text{Min}(Y_i \mid C_i > 0) + 1,$$

or the year of the most recent publication with at least one citation minus the year of the earliest publication with at least one citation (plus one to consider the time spent preparing the earliest publication). For most active researchers, this will likely be simply the number of years since they first published since it will most often be the difference in age between their first publication (which probably has at least one citation) and their most recent publication to get a single citation (probably published within the last year or two).

The **citation speed index** (Bornmann and Daniel 2010) is meant to be a complement to the h-index. Instead of measuring the number of citations, it is a measure of the speed at which the first citation for a publication accrued. If M_i is the number of months since the first citation for the i^{th} publication (ranked by M_i), a researcher has citation speed index of s if s of their papers were cited at least s months ago, or

$$s = \max_i \left(i \le M_i\right).$$

This index is somewhat more difficult to calculate since it requires knowing the month of the first citation for each publication. Its usefulness as a general career-level measure is somewhat questionable; its primary use might be to compare sets of publications published from the same year (e.g., if comparing the papers published in 2000 among two different researchers, the index would indicate which researcher was cited most rapidly).

Liang (2006) suggested calculating the *h*-index for a series of increasing time intervals (which he called the ***h*-index sequence**), starting concurrently and moving backwards in time; thus the first index might represent the last two years, the second the last three years, the third the last four years, etc. This is essentially the reverse of the type of data which I've used throughout this manuscript where I started with a time point in the past (1997, my first year of publication) and then kept increased the time interval by a year. Liang's approach might reveal the contemporary nature of the index change; my measures were meant to reflect the change or stability of the indices across time and not designed as a general approach for evaluating researchers.

Table 12. Time adjusted impact factor indices.

Date	1997	1998	1999	2000	2001	2002	2003	2004
Hirsch *m*-quotient	n/a	2.00	1.50	1.67	1.50	1.40	1.50	1.57
ar-index	1.00	3.00	4.18	4.90	7.61	9.01	10.10	12.31
dynamic *h*-type-index	n/a	4.40	6.63	10.73	14.94	18.95	24.46	33.02
hpd-index	2	5	5	7	10	13	15	17
contemporary *h*-index	2	3	3	5	6	9	10	10
trend *h*-index	2	4	5	7	8	12	14	17
Impact vitality	n/a	n/a	n/a	n/a	1.93	1.82	1.59	1.64
specific-impact *s*-index	0.00	2.73	3.59	4.20	5.35	6.53	7.15	8.57
f-index	1	2	2	4	5	6	7	8

Date	2005	2006	2007	2008	2009	2010	2011
Hirsch *m*-quotient	1.63	1.67	1.70	1.73	1.92	1.85	1.71
ar-index	13.31	14.85	15.46	16.38	17.81	17.92	18.71
dynamic *h*-type-index	41.93	51.64	60.78	70.42	83.89	93.40	98.48
hpd-index	20	24	23	25	24	28	29
contemporary *h*-index	12	14	16	17	18	20	20
trend *h*-index	17	20	23	24	25	28	29
Impact vitality	1.48	1.41	1.26	1.18	1.16	1.13	1.06
specific-impact *s*-index	9.77	10.91	11.79	12.68	13.75	14.70	15.03
f-index	9	10	11	11	11	14	15

Unlike most of the other impact factors, the majority of the time adjusted indices can decrease through time. As with many of the other core-description metrics, the *ar*-index and the dynamic *h*-type-index are somewhat difficult to interpret. The *hpd*-index and contemporary *h*-index both do essentially the same thing, using average citations per time period (either 10 years or 4 years, although any time period is possible), projecting citation counts forward for those publications whose window is less than the specified time period. Their interpretation is straightforward as long as one is cognizant of the time window over which each is measured. As mentioned earlier, trend *h*-index and impact vitality are both difficult to calculate, and impact vitality in particular is potentially overly sensitive to extremely recent citations. The specific-impact *s*-index requires a number of assumptions missing from the other metrics, making it a more questionable and difficult to interpret measure. As already discussed, the *f*-index may functionally simply be the time period over which someone has been publishing, making it a metric of limited usefulness.

Miscellaneous Indices

Beyond all of the indices already discussed, many other indices have been proposed which do not fall into any of the mentioned categories. Generally, these require data or information not readily available and are much more difficult to calculate than the standard indices already discussed. Many are briefly summarized for completeness.

Vaidya (2005) proposed adjusting the *h*-index for the proportion of an author's time dedicated to research, arguing that a researcher with 100% of their time (FTE) dedicated to research should not be directly compared to one with only 40% of their time dedicated to research. Vaidya's ***v*-index** also includes a simple adjustment for the age of an author, since it is based on the slope of change (Hirsch's *m*-index) of the *h*-index through time. If *p* is the proportion of time dedicated to research, the value is simply

$$v = \frac{m}{p} = \frac{h}{p\left(Y_{Now} - Y_0\right)}$$

Although differential workload has largely been ignored within the impact index community, this basic approach could be applied to any index if one felt the need to make such an adjustment.

The **second generation *h*-index** (Kosmulski 2010b; Schubert 2009) take citation chains to the next level; instead of looking at the number of citations for a researcher's publications (the 1st generation citations), it looks at how many citations each of the citing publications has (the

2nd generation citations). Again, this index requires much more complicated data collection in order to calculate.

The ***n*-index** (Namazi and Fallahzadeh 2010) is designed to standardize the *h*-index across different disciplines by dividing *h* by the top impact factor of the journals in a researcher's field. The essentially identical correction had previously been suggested by Iglesias and Pecharromán (2007) except using the average impact factor of journals in the researcher's field.

The ***ch*-index** (Ajiferuke and Wolfram 2010) uses the number of citers rather than the number of citations to measure impact (that is, if a single author cites a paper 10 times, that only counts as 1 citer). Once again, this is somewhat more difficult to calculate because it requires that one determine individual citers rather than just total citations. The ***f*-index** (Katsaros *et al.* 2009) (different than the previously mentioned *f*-index) is a much more complicated, weighted approach to the same issue of counting citers rather than citations.

The **h_{int}-index** (Kosmulski 2010a) attempts to measure international recognition in an *h*-index manner by counting the number of countries which cited each publication rather than the total number of citations.

A series of indices have recently been described to measure collaboration impact or effectiveness. The **researcher collaboration (RC-index)** and **community collaboration (CC-index) indices** (Abbasi *et al.* 2010) are designed to measure the quantity and quality of co-authorships of individuals and groups of individuals, respectively. They are difficult to calculate because they require information on authorship overlap across publications as well as information on the quality (individualized impact) of both collaborators. The **φ-index** (Schubert 2012) is constructed identically to the *h*-index, except that it measures the largest number of coauthors, φ, with which a researcher has published at least φ publications. (My φ-index as a I write this is 4, meaning I've published at least 4 papers with each of 4 different coauthors, but do not have at least 5 publications with each of different coauthors). The ***d*-index** (Di Caro *et al.* In press) attempts to measure the dependence among coauthors by estimating the influence of a specific one coauthor on the productivity of another.

Conclusion

Despite many flaws leading to a huge literature on impact factors, the *h*-index has some major advantages over many of its alternates, particularly with respect to ease of calculation and interpretation. Alternate approaches to defining core publication, such as the *g*-index, may have certain advantages over *h* with respect to stability and better capturing of the citation distribution. Metrics which account for excess citations (such as the tapered *h*-index) are a bit more difficult to calculate, but better describe the overall citation distribution and may well serve to distinguish between researchers with identical *h*. Corrections for self-citations are possible, but likely only a problem for those with low impact factors. There are many approaches to dealing with publications with multiple authors, but no single approach is clearly superior to the others and different metric choices may lead to very different results. Many of the alternate impact metrics are difficult to interpret, while many others require non-trivial data collection making them impractical for general use.

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